PRELIMINARY DESIGN OF COMPOSITE CATENARY RISERS USING OPTIMIZATION TECHNIQUES

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Abstract. The use of steel risers for deepwater application is not always feasible, due to its high weight. Fiber reinforced composite materials offer an interesting alternative due to its several advantages, such as high specific strength and stiffness, high corrosion resistance, low thermal conductivity, good structural damping, and high fatigue resistance. Thus, the use of composite risers is an interesting alternative to deepwater oil fields. The design of composite risers requires the consideration of different load conditions and lamination schemes. Therefore, the design of a composite riser joint is more complex than conventional risers since strength and stiffness of these components depend on the number of layers and the thickness and orientation of each layer. This work presents a methodology for design of Composite Catenary Risers (CCR) based on the procedure used for design of conventional risers. An optimization technique is applied to the preliminary design of composite riser joints in order to minimize the cost of joint, assumed proportional to the volume of material. The design variables are the thickness and orientation of each layer. Strength and stability constraints are considered in the optimization model. Simple and efficient expressions, based on the Classical Lamination Theory, are used for stress computation and stability analysis. Multiple load cases are included. The proposed methodology is applied to the preliminary design of CCRs with different water depths, liner materials, and failure criteria.
1 INTRODUCTION

The depletion of existing reserves and the increasing demand for oil and gas has led to the search for deepwater fields and research in new technologies to make the production feasible. The use of steel risers for deepwater applications may be compromised due to its high weight. Fiber reinforced composite materials offer an alternative due to its several advantages, such as high specific strength and stiffness, high corrosion resistance, low thermal conductivity, excellent damping properties and fatigue resistance. Thus, the use of composite risers is an interesting alternative to deepwater oil fields.

These favorable properties have motivated the oil industry to use composite materials in some offshore applications, such as risers and tethers (Ochoa and Salama, 2005). The reduced weight obtained by the use of composite material risers in replacement to steel risers can be substantial, leading to a significant reduction of top tension requirements, which allows the use of simpler and smaller tension mechanisms and smaller platforms (Tamarelle and Sparks, 1987; Salama, 1997; Ochoa and Salama, 2005).

It is important to note that the literature about composite risers deals almost exclusively with Top Tensioned Risers for Tension Leg Platforms (TLPs). On the other hand, this work presents a methodology for Composite Catenary Riser (CCR) design, based on the methodology used for conventional risers.

In this procedure, the structural analysis, including the dynamic and nonlinear effects, is carried-out using the global-local approach. This approach, also widely adopted for flexible risers (Witz, 1996), allows the use of standard 3D beam elements for global analysis provided that the equivalent mechanical properties of the composite cross-section are evaluated properly. After the global analysis, critical riser joints are selected and analyzed using refined local finite element models to evaluate the stresses and strains in each ply due to internal or external pressure and internal forces (axial force, bending and torsional moments) computed in the global analysis.

The design of laminated composite structures is a complex task, since the strength and stiffness of laminates depend on the number of layers and the material, thickness, and fiber orientation of each layer. Thus, the use of the conventional trial-and-error strategy is not adequate and the natural approach is to apply optimization techniques. However, since several loading and environmental conditions need to be considered in the design, it is not feasible yet to apply an optimization technique to the detailed design of composite risers due to the high computational cost of the required structural analyses. Therefore, optimization techniques will be applied in this work only in the preliminary design of composite riser joints.

The design variables are thickness and orientation of each layer, as the number of layers and the material are defined a priori. The objective is to minimize the cost of the riser joint, assumed to be proportional to the volume of composite material. Strength and stability constraints are included and different load cases (e.g. installation and operation) may be considered in the optimization model. As the objective is only to obtain a preliminary design, the axial force is estimated using the catenary equations and the stresses and buckling loads are evaluated using the Classical Lamination Theory (CLT).

This paper is organized as follows. Section 2 discusses the mechanical behavior of composite materials, including the Classical Lamination Theory and failure criteria. Section 3 discusses the analysis and design of composite risers. Section 4 presents a simple preliminary design methodology. Section 5 describes the optimization model used for preliminary design, while Section 0 presents some numerical examples. Finally, Section 7 presents the main conclusions.
2 FIBER REINFORCED COMPOSITES

Composite materials are formed by two or more materials combined in a macroscopic scale (Reddy, 2004). Fiber Reinforcement Composites (FRC) are composed of high strength and stiffness fibers (e.g. glass and carbon fibers) embedded in a polymeric matrix, as epoxy resin (Jones, 1999). Thus, FRC behave macroscopically as orthotropic materials, where principal directions are parallel and perpendicular to fibers.

2.1 Classical Lamination Theory

Laminated composites are constituted by series of layers (also know as plies or laminas) with different properties, assumed perfectly bonded to each other. In the case of fiber reinforced composites, each lamina behaves macroscopically as a homogeneous and orthotropic material in the coordinate system of material \( (x_1, x_2, x_3) \), where \( x_1 \) is parallel to the fibers, \( x_2 \) is perpendicular to the fibers and \( x_3 \) is perpendicular to the lamina (see Figure 1).

Figure 1 – Local (material) and global coordinate systems.

It can be verified experimentally that mechanical behavior of fibrous composites can be considered as linear elastic until close to its rupture (Jones, 1999). In this case, the stress-strain behavior may be represented by generalized Hooke’s Law (Jones, 1999; Reddy, 2004). A Plane Stress state is assumed to prevail in each ply and the relation between stress \( (\sigma_1) \) and strain \( (\varepsilon_1) \) in the material (local) system is given by

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_6
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_6
\end{bmatrix}
\Rightarrow \quad \sigma_i = Q \varepsilon_i
\]  

(1)

where the coefficients of the constitutive matrix \( Q \) are given by

\[
Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} ; \quad Q_{12} = \nu_{12}E_2 ; \quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} ; \quad Q_{66} = G_{12}
\]  

(2)

The equations that govern the solution of the problem are written in the global coordinate system \((x, y, z)\), while the constitutive equation is given in the material system. Thus, it is important to transform stresses and strains between these two systems. The strains can be transformed from the global to the local system using the equation

\[
\varepsilon_i = T \varepsilon
\]  

(3)

where \( T \) is the transformation matrix computed from the director cosines of the local axes with respect to the global axes (Cook et al., 2002). Using the Virtual Work Principle, it may be shown (Cook et al., 2002) that the stress-strain relation in the global system is given by
\[ \sigma = \overline{Q} \varepsilon, \quad \text{with} \quad \overline{Q} = T^T Q T, \]  

where \( \overline{Q} \) is called the transformed constitutive matrix. The coefficients of the transformed stiffness matrix are given by

\begin{align*}
\overline{Q}_{11} &= \cos^4 \theta \big( Q_{11} + 2 \sin^2 \theta \cos^2 \theta Q_{12} + 2 Q_{66} \big) + \sin^4 \theta Q_{22} \\
\overline{Q}_{12} &= (Q_{11} + Q_{12} - 4 Q_{66}) \sin^2 \theta \cos^2 \theta + (\sin^4 \theta + \cos^4 \theta) Q_{12} \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2 Q_{66}) \sin \theta \cos^5 \theta + (Q_{12} - Q_{22} + 2 Q_{66}) \sin^3 \theta \cos \theta \\
\overline{Q}_{22} &= \sin^4 \theta Q_{11} + 2 (Q_{12} + 2 Q_{66}) \sin^2 \theta \cos^2 \theta + \cos^4 \theta Q_{22} \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2 Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2 Q_{66}) \sin \theta \cos^3 \theta \\
\overline{Q}_{66} &= (Q_{11} + Q_{12} - 2 Q_{66}) \sin^2 \theta \cos^2 \theta + (\sin^4 \theta + \cos^4 \theta) Q_{66}
\end{align*}

(5)

According to the Classical Lamination Theory (Jones, 1999; Reddy, 2004), the strains at planes parallel to the middle surface of laminate are given by

\[ \varepsilon = \varepsilon_m + z \kappa \]

(6)

where \( \varepsilon_m \) represents the membrane strains (strains at the mid-surface) and \( \kappa \) represents the curvatures of the laminate. These parameters are also known as generalized strains. It is important to note that in the case of plates and shells it is more convenient to work with stress resultants (forces and moments) than stresses. The resultant forces and moments (also known as generalized stresses) are obtained by integration of stresses through the laminate thickness \( t \):

\begin{align*}
N_x &= \int_{-l/2}^{l/2} \begin{pmatrix} \sigma_x \\ \tau_{xy} \end{pmatrix} dz, \\
N_y &= \int_{-l/2}^{l/2} \begin{pmatrix} \sigma_y \\ \tau_{xy} \end{pmatrix} dz, \\
M_x &= \int_{-l/2}^{l/2} \begin{pmatrix} \tau_{xy} \\ \tau_{yx} \end{pmatrix} z dz, \\
M_y &= \int_{-l/2}^{l/2} \begin{pmatrix} \tau_{xy} \\ \tau_{yx} \end{pmatrix} z dz.
\end{align*}

(7)

Using Eqs. (4), (6), and (7), the generalized stress-strain relationship can be written as:

\[ \begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon_m \\ \kappa \end{bmatrix} \]

(8)

where \( A \) is the membrane (or extension) stiffness matrix, \( D \) is the bending stiffness matrix, and \( B \) is the bending-extension coupling stiffness. The coefficients of these matrices can be computed using Eqs. (4), (6), and (7):

\begin{align*}
A_{ij} &= \sum_{l=1}^{n} \overline{Q}_{ij} (z_{k+1} - z_k) \\
B_{ij} &= \sum_{l=1}^{n} \overline{Q}_{ij} \left( \frac{z_{k+1}^2 - z_k^2}{2} \right) \\
D_{ij} &= \sum_{l=1}^{n} \overline{Q}_{ij} \left( \frac{z_{k+1}^3 - z_k^3}{3} \right)
\end{align*}

(9)

where \( z_{k+1} \) and \( z_k \) are the coordinates of the top and bottom of each ply and \( n \) is the number of plies.

2.2 Failure Criteria

The stresses and strains in the material coordinate system can be used to evaluate the structural safety of the laminate, using an appropriate failure criterion. For this purpose, the stresses or strains are compared to those of a failure state obtained in laboratory. For isotropic materials, whose mechanical properties are constant in every direction, classic failure criteria
such as Tresca and von Mises are extensively used. For such materials, only one strength parameter, defined by a yield or failure stress, is used and there are few failure modes.

On the other hand, for orthotropic materials, the mechanical properties vary in the three main orthogonal directions. Thus, different strength parameters have to be used for each direction. Furthermore, for fiber reinforced composites, such parameters have different values for tension and compression, with 9 independent parameters in a three-dimensional stress state and 5 for plane stress. The failure criteria of orthotropic materials can be applied to detect the failure of individual layers.

Laminate failure is more difficult to define than ply failure. Two criteria have been used: First Ply Failure (FPF) and Ultimate Laminate Failure (ULF). FPF considers that the laminate fails when the first ply fails. On the other hand, in ULF a progressive failure analysis is performed, in which the load is applied in a sequence of steps and the stiffness of failed plies is degraded. The load is increased until the last ply fails leading to the ultimate load.

In the present work, the First Ply Failure is adopted as the criterion to define laminate failure and two well-known criteria, the Maximum Stress and the Tsai-Wu (Jones, 1999; Daniel and Ishai, 2006), are used to detect ply failure.

### 2.3 Maximum Stress Criterion

In the Maximum Stress criterion, the stresses in each direction of the material system are compared separately with the strength in that direction. Therefore, there is no interaction between the stresses in different directions. In a two-dimensional analysis, the failure envelope for this criterion is represented by:

\[
\begin{align*}
\sigma_1 &= \begin{cases} 
F_{1t} & \text{when } \sigma_1 > 0 \\
-F_{1c} & \text{when } \sigma_1 < 0 
\end{cases} \\
\sigma_2 &= \begin{cases} 
F_{2t} & \text{when } \sigma_2 > 0 \\
-F_{2c} & \text{when } \sigma_2 < 0 
\end{cases} \\
\tau_6 &= F_6
\end{align*}
\]

where \(F_{1t}, F_{1c}, F_{2t}, F_{2c}\) are the in-plane tensile (\(t\)) and compression (\(c\)) strengths and \(F_6\) is the in-plane shear strength. The Safety Factor (\(SF\)) is determined as the minimum relation between the material strength and the corresponding stress component:

\[
SF = \min \left\{ \begin{array}{ll}
\frac{F_{1t}}{\sigma_1} & \text{when } \sigma_1 > 0 \text{ and } -\frac{F_{1c}}{\sigma_1} \text{ otherwise} \\
\frac{F_{2t}}{\sigma_2} & \text{when } \sigma_2 > 0 \text{ and } -\frac{F_{2c}}{\sigma_2} \text{ otherwise} \\
\frac{F_6}{\tau_6} & 
\end{array} \right. 
\]

The Maximum Stress Criterion is very simple, intuitive, and easily implemented. Moreover, it allows the consideration of different strength parameters for tensile and compressive strengths and the determination of the failure mode.
2.4 Tsai-Wu Criterion

The Tsai-Wu criterion is based on the polynomial failure theory proposed by Gol'denblat and Kopnov (Daniel and Ishai, 2006). The Tsai-Wu failure surface for plane stress is given by

\[ f_1 \sigma_1^2 + f_2 \sigma_2^2 + f_{11} \sigma_1^2 + f_{22} \sigma_2^2 + 2 f_{12} \sigma_1 \sigma_2 + f_{66} \sigma_6^2 = 1 \]  

(12)

The \( f \) parameters are obtained experimentally through uniaxial stress tests in each direction. The parameters used in this work are as follows (Daniel and Ishai, 2006):

\[ f_1 = \frac{1}{F_{1r}} - \frac{1}{F_{1c}} \; ; \; f_2 = \frac{1}{F_{2r}} - \frac{1}{F_{2c}} \; ; \; f_{66} = \frac{1}{F_6} \]

\[ f_{11} = \frac{1}{F_{1r} F_{1c}} \; ; \; f_{22} = \frac{1}{F_{2r} F_{2c}} \; ; \; f_{12} = -\frac{1}{2} \sqrt{f_{11} f_{22}} \]

(13)

The Safety Factor is the value that multiplies the stress vector \( (\sigma_1) \) in order to satisfy Eq. (12). The resultant expression is a second-degree polynomial equation, whose positive root is the Safety Factor:

\[ SF = -b + \sqrt{b^2 - 4a} \]

(14)

where

\[ a = f_1 \sigma_1^2 + f_2 \sigma_2^2 + f_{66} \sigma_6^2 + 2 f_{12} \sigma_1 \sigma_2 \]

\[ b = f_1 \sigma_1 + f_2 \sigma_2 \]

(15)

Tsai-Wu criterion fits well the available experimental data (Barbero, 1999). In addition it is simple and easy to implement. Furthermore, the linear terms of Eq. (12) allow the consideration of the difference between tensile and compressive strengths. On the other hand, it does not indicate the failure mode.

3 ANALYSIS AND DESIGN OF COMPOSITE CATEYRIS RISERS

Rigid risers are assembled using a series of steel or composite joints connected to each other by appropriate (e.g. pin and box or flange) connections. The length of each riser joint generally varies from 12 m to 24 m. Composite riser joints are composed generally by three elements: liner, composite tube and terminations (ABS, 2006; Ochoa, 2006), as depicted in Figure 2.

![Figure 2 – Element of a composite riser joint (adapted from Smith and Leveque, 2005).](image)

The liners are responsible for fluid containment, ensuring the tightness of riser and
avoiding leaking and loss of pressure. These elements are necessary since composite materials are porous and subjected to micro-cracks. Most composite joints have an inner and an outer liner. Inner liners can be elastomeric, thermoplastic or metallic (steel or titanium). In the selection of the inner line material, in addition to fluid tightness other criterion should be observed, as cost, adhesiveness to composite and metallic termination, abrasion and corrosion resistance to the reservoir fluid and impact resistance to mechanical tools inside drilling risers (Tamarelle and Sparks, 1987; ABS, 2006; Ochoa, 2006). The inner liner can also be used as a mandrel during the joint manufacture.

The elastomeric liners are synthetic rubbers, such as Buna-N and HNBR, which have excellent properties, including impact resistance and corrosion resistance and high temperature. The synthetic rubber may be used as inner liner in applications where the tube is not in permanent contact with oil and gas (Tamarelle and Sparks, 1987). In the case of drilling risers, the metallic liners are preferred due to damage problems caused by the impact of drilling tools (Salama et al., 2002). Metallic liners can also have structural function, contributing to the resistance to internal pressure (burst) and collapse due to external pressure.

Generally, the outer liner is constituted of synthetic rubber or thermoplastics. The main function of the outer liner is fluid (sea water) containment and protection against external impact and gouging (Kim, 2007; Ochoa, 2006). The requirements of the material for the outer liner includes: corrosion resistance due to direct contact with sea water, impact resistance, and resistance to environment effects, as temperature and ultra-violet radiation (Tamarelle and Sparks, 1987; Ochoa, 2006). An additional external layer can be used as a mechanical protection from impacts during transportation and handling.

The composite tube is the main structural element of the riser joint. It is constituted by various layers (or plies) of fibrous composite materials. Carbon and glass fibers are the most used in composite risers, while the matrix is usually a polymeric resin (e.g. epoxy). The glass fibers have lower strength and stiffness, but lower cost and higher impact resistance, when compared to carbon fibers.

The material, thickness and fiber orientation of each layer should be chosen in order to provide sufficient strength and stiffness to the riser joint. The selection of the composite lay-up for a given set of external loads is a very difficult task. A general guideline is to put the fibers along the load paths. Thus, fibers can be oriented in circumferential direction to resist internal and external pressure and in axial direction to resist the axial force and bending moment.

In a cross-ply lay-up, the fibers in the axial layers should be aligned as much as possible with the global (longitudinal) axis of the riser, while the hoop layers should be close to 90° with respect to the global axis of riser. Axial layers oriented at angles of ±20° have also been used in the design of composite risers (Tamarelle and Sparks, 1987; Sparks et al., 1988, Meniconi et al., 2001).

The joint terminations are metallic pieces, normally of steel or titanium, composed of two parts: the Metal-to-Composite Interface (MCI) and the connections. The MCI transmits the stresses from the composite tube to the metallic connectors, while the connections are responsible to join the composite joints, assembling the riser. Pin and box or flange connections have been used (Salama et al., 2002; Smith and leveque, 2005). The MCI and the connectors can be fabricated as only one piece, eliminating welding and the fatigue problems associated.

3.1 Design methodology

A methodology for the design of composite catenary risers is presented and discussed in
the following. This work focuses only on the design of the composite tube and the design of the terminations (MCI/connections) is not addressed here.

The design process begins with the definition of the riser requirements, including the configuration and geometric parameters, installation and operation conditions, floater data, environmental conditions, etc.

Figure 3 shows a flowchart of the proposed design methodology. The first step is the conceptual design, which involves the definition of materials for each element of composite riser joint: composite tube, inner and outer liners, and mechanical protection layer.

The preliminary design consists in the definition of the size (thickness) of the non-structural elements (liners and protection layer) and the determination of an initial lay-up for the composite tube. The proposed laminate should resist to the internal and external pressure associated with each design scenarios (load cases) considered. Typically, these load cases include installation (empty riser) and operation (riser with internal fluid).

At the end of preliminary design the materials, dimensions, and stack-sequence of the composite joint are known and the weights (dry and wet) and mechanical properties can be computed. Therefore, the structural analysis of the riser can be carried-out for the different load cases.

Composite structures are generally analyzed by the Finite Element Method (FEM) using solid or shell elements, since these elements allow the consideration of the material, thickness, and orientation of each ply. The use of shell or solid elements yields the stresses and strains in each ply, which are used to verify the design safety using an appropriate failure criterion. However, the modeling of risers using solids or shells elements leads to a prohibitively high computational cost for practical applications. Thus, the analysis of riser is carried-out in two levels: global and local, as depicted in Figure 4. This approach is generally used for flexible risers (Witz, 1996) and has also been recommended for composite risers.
In this approach, the global analysis of the riser is carried-out using beam elements due to the large riser length. These elements allow the consideration of dynamic and nonlinear effects in a simple and efficient way.

In order to the beam model produces accurate results it is necessary that the equivalent mechanical properties \((EA, EI, GJ)\) of the riser are appropriately computed. It is important to note that the computation of the cross-sectional properties of a composite riser is much more complex than for metallic risers, since the stiffness (e.g. \(EI\)) of one laminate cannot be calculated simply multiplying the material properties (e.g. \(E\)) by geometrical properties (e.g. \(I\)). This happens because laminate sections are neither homogeneous nor isotropic, as they are composed of several orthotropic layers.

Melo et al. (2009) presented different methods for the computation of the bending stiffness \((EI)\) of composite tubes and compared the results obtained using these procedures with the results obtained by using a 3D finite element model. Good results for most lay-ups were obtained using the homogenization by the Classical Lamination Theory. Other methods lead to better results, but are more complex to implement. Due to its simplicity, homogenization has been used in the computation of the mechanical properties of composite risers (Kim, 2007).

The global analysis yields the displacements and stresses resultants (axial force, bending and torsional moments) along the riser. However, the use of beam elements do not allow the direct computation of stresses and strains at each ply of the composite tube.

Therefore, the stress resultants calculated in global analysis at critical joints are used as input data for local analysis, as shown in Figure 4. The local analysis is carried-out using refined shell or solid finite element models, since these elements allow the computation of stresses and strains at each ply, as required by the failure criteria of composite materials.

Finally, the Safety Factor of the proposed joint is computed using an appropriate failure criterion (e.g. Maximum Stress, Maximum Strain or Tsai-Wu). If the Safety Factor is satisfactory the proposed joint can be accepted, otherwise a new lay-up should be proposed. The joints are analyzed and verified until a satisfactory design is found. A final step, not shown in Figure 3, is the verification of fatigue life, which can be performed using the techniques presented in Kim (2005; 2007).
4 PRELIMINARY DESIGN

Starting from the riser specifications (e.g. sea depth, top angle, installation and operation conditions), the aim of preliminary design is to define an initial composite riser joint. This initial joint will be analyzed, verified, and refined in order to satisfy the prescribed safety requirements in an economical way.

The main difficulty in the definition of an initial joint design is that the internal forces in the riser depend on applied loads, including self-weight and buoyancy, or mechanical properties of the riser that cannot be evaluated unless the materials, geometry, and lay-up of the joint are already known. In order to overcome this problem a simple iterative methodology to the preliminary design of composite joints of catenary risers is presented in Figure 5.

![Figure 5 – Preliminary design.](image)

The first step includes the sizing of the non-structural parts (inner and outer liner, mechanical protection layer, and thermal coating) and the definition of the thickness of hoop and axial layers. The thickness of inner and outer liners and the protection layer are defined based on previous experience.

The evaluation of thickness of the thermal coating depends on the riser insulation requirements and will not be addressed here. However, it should be noted, that one of the advantages of fiber reinforced composite materials with polymeric matrix is their lower thermal conductivity when compared with metals, which allow the use of a smaller thermal coating thickness.

Thereafter, the composite lay-up of the tube can be determined by defining the number of plies and thickness and orientation of each ply. The initial lay-up should have sufficient burst and collapse strength, resisting the pressure differentials (internal or external) for all design scenarios. The maximum expected internal pressure depends on the technical specifications of the riser, while the maximum external pressure depend on the sea depth. An appropriate load factor (γ_F) should be used to obtain the design external pressure. Table 5-2 of DNV-RP-F201 (2001) suggests γ_F = 1.1.

Adopting a cross-ply lay-up, it is necessary to define the thickness of hoop (90°) and axial (0°) layers. Neglecting the biaxial behavior of the composite and the contribution of the liners, the thickness (t_{90}) of hoop can be estimated from the stresses in thin-walled tubes:
where \( p \) is the internal pressure differential, \( R \) is the riser radius and \( t \) is the thickness. The thickness of axial layers \((t_0)\) can be estimated considering the end-forces due to internal pressure:

\[
\sigma_x = \frac{pR}{2t} \Rightarrow t_0 \geq \frac{pR}{2F_{lt}} \quad (17)
\]

The definition of layer thickness for angle-ply lay-ups is much more difficult, since the biaxial behavior cannot be neglected.

An initial lay-up can be proposed using the hoop and axial total thickness computed using Eq. (16) and (17), respectively. This laminate should resist the buckling collapse due to the external pressure (hoop buckling), as will be discussed in Section 4.2. Due to the uncertainties about the material behavior and simplifications made in the buckling analysis, a Safety Factor of 3.0 was adopted for buckling collapse. This value is recommended as the collapse resistance in qualification tests of composite riser joints (ABS, 2006). The contribution of the liners to the buckling strength may be neglected, as a conservative solution.

Once obtained a laminate with sufficient buckling pressure, the strength of the joint to internal forces (axial force, bending and torsional moments) should be evaluated. These internal forces can be obtained from a global finite element analysis of the riser, considering dynamic and nonlinear effects due to external loads, floater offset, currents, and waves, as discussed in Section 3. However, this procedure is not adequate to the preliminary design due to the high computational cost of a nonlinear dynamic analysis.

Therefore, the axial forces are estimated in this work using the catenary equations. This approach allows the consideration of self-weight, internal fluid, buoyancy, and floater offset. On the other hand, environmental loads, bending and dynamic effects are not included. In order to consider these effects in a simple way, axial forces computed using the catenary equations are multiplied by Dynamic Amplification Factor \((DAF \geq 1.5)\).

The thicknesses of axial layers depend on the riser tension, which depends on the riser wet weight. However, the wet weight depends on the thickness of the riser joint. Therefore, the determination of the thicknesses of axial plies is an iterative process. In order to obtain the weight of a joint, the length of terminations (MCI + connections) should be considered. At a preliminary design the length of terminations can be assumed as 10-20% of the joint length.

The initial configuration and the effective axial force \((N_{ef})\) are estimated from the riser wet weight, mean top angle, floater offset, and water depth using the catenary equations. The true axial force \((N_{tw})\), i.e. the resultant of axial stresses in the riser wall, is given by (Sparks, 2007):

\[
N_{tw} = N_{ef} + p_e A_e - p_i A_i
\]

where \( p_e \) is external pressure, \( A_e \) is external area, \( p_i \) is internal pressure, and \( A_i \) is the internal area. The design force is obtained using the appropriate load factor \((N_d = \gamma_N \cdot N)\).

The ply stresses due to internal pressure, external pressure, and true axial force can be accurately computed using an axisymmetric finite element formulation (Rocha et al, 2009). This approach leads to very accurate results, but requires a careful mesh generation. A more simple and efficient alternative is to use the Classical Lamination Theory. It is important to note that the steel liner should be included in the analysis in order to allow the verification of
the safety of this element. The Tsai-Wu failure criterion (Daniel and Ishai, 2006) is used to evaluate the Safety Factor of composite plies, while the von Mises criterion is applied to the steel liner (if present). The concept of First Ply Failure (FPF) is used to define laminate failure. Thus, the failure of the laminate occurs when the first ply fails (Daniel and Ishai, 2006). Since composite materials behave elastically until very close to failure, the FPF criterion allows the use of elastic stress analysis in the verification of riser strength.

The required Safety Factors depend on the material and analysis method, with large values adopted for the composite due to the higher dispersion of experimental results. The values adopted in this work are presented in Table 1 (DNV, 2003a; DNV, 2003b).

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Steel</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simplified</td>
<td>1.50</td>
<td>3.0</td>
</tr>
<tr>
<td>FEM</td>
<td>1.25</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 1: Required resistance factors.

If the safety factors are greater than the required resistance factors, the laminate can be accepted, otherwise a new lay-up should be proposed. It should be noted that the procedure proposed in Figure 5 is based on a trial-and-error approach. Due to the large number of parameters to define (number of plies, thickness and orientation of each ply) and the lack of guidance about the necessary changes in these parameters, it is very difficult to find a laminate meeting the strength and stability requirements in an economical way.

4.1 Stresses in Thin-Walled Laminated Risers

The stresses in thin-walled laminated composite tubes can be computed with good accuracy using the Classical Lamination Theory. For thin-walled tubes subjected only to axisymmetric loads, there is no bending \((M = 0)\) and the in-plane (membrane forces) are given by

\[
N_x = \frac{N_{nw}}{2\pi R} \\
N_y = p_x R - p_y R_e \\
N_{xy} = \frac{T}{2\pi R^2}
\]

where \(T\) is the torsional moment. Only symmetric lay-ups will be adopted in this work. For these lay-ups there is no extension-bending coupling, as \(B = 0\). Thus:

\[
\begin{bmatrix} N_m \\ 0 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} \varepsilon_m \\ \kappa \end{bmatrix} \Rightarrow \begin{bmatrix} \varepsilon_m = A^{-1} N_m \\ \kappa = 0 \end{bmatrix}
\]

Therefore, the strains in the global system are constant throughout the laminate thickness and Eq. (6) simplifies to

\[
\varepsilon = \varepsilon_m
\]

The strains in each ply are transformed to the material system using Eq. (3) and the stresses in this system are computed using Eq. (1).
4.2 Stability of Composite Risers

Rigid risers can be subjected to high compressive stresses during installation or operation. It is well known that structures subjected to compressive stresses may collapse due to loss of stability (Bazant and Cedolin, 1991). Hoop buckling is the local buckling of the tube wall (shell buckling) caused by the compressive stresses generated by external pressure. Hoop buckling is a major concern for deepwater risers and pipelines due to the high hydrostatic pressure, especially when the riser is empty and there is no internal pressure (e.g. during riser installation).

The buckling loads of cylindrical shells depend on the length of tube \( L \), mean radius \( R \), wall thickness \( t \), material parameters and boundary conditions. The effect of the geometry and material on stability of isotropic cylindrical shells can be studied through of curvature or Batdorf parameter \( Z \) (Batdorf, 1947; Brush and Almroth, 1975):

\[
Z = \left( \frac{L}{R} \right)^2 \left( \frac{R}{t} \right)^{1/2} \sqrt{1 - \nu^2}
\]

where \( \nu \) is the Poisson’s ratio of the material. For long cylindrical shells of isotropic materials, the elastic buckling pressure is given by:

\[
P_{cr} = \frac{E}{4(1-\nu^2)} \left( \frac{t}{R} \right)^3
\]

Higher critical pressures are obtained for short cylinders due to the effect of the restrained ends.

It is well-known that the unavoidable geometric imperfections reduce the load carrying capacity of compressed shells due to the geometric nonlinear effects. For design purposes the collapse pressure \( p_{col} \) is obtained by the following expression:

\[
p_{col} = k_p P_{cr}
\]

where \( k_p \) is a reduction factor to account for the effects of the geometric imperfections. Weingarten et al. (1968) recommend to consider \( k_p = 0.75 \) for short cylindrical shells and \( k_p = 0.9 \) for long ones.

According to Weingarten et al. (1968) and DNV (2003b), the critical pressure of long orthotropic cylindrical shells is given by

\[
P_{cr} = \frac{3}{R^3} \left( D_{22} - \frac{B_{22}^2}{A_{22}} \right)
\]

where \( A_{22}, B_{22}, \) and \( D_{22} \) are computed according to Eq. (9). It is interesting to note that for isotropic materials \( B_{ij} = 0 \) and \( D_{22} = E t^4/12(1-\nu^2) \), therefore Eq. (25) reduces to Eq. (23). A knock-down factor \( k_p = 0.75 \) is recommended for design purposes.

The classification of short or long shells can be made through the parameter:

\[
\bar{Z} = \left( \frac{D_{22}}{D_{11}} \right)^{3/2} \left[ \frac{\left(A_{11}A_{22} - A_{12}^2\right)}{12A_{22}D_{11}} \right]^{1/2} \left( \frac{L}{R} \right)^{1/2} \left( \frac{R}{t} \right)
\]

For isotropic materials \( E_1 = E_2 = E \) and \( \nu_{12} = \nu_{21} = \nu \) the \( \bar{Z} \) parameter reduces to \( Z \). Therefore, \( \bar{Z} \) parameter can be interpreted as the curvature parameter for orthotropic...
Due to the complexity of the problem, the study of stability of shells requires the utilization of an adequate numerical method. The Finite Element Method (Bathe, 1996; Cook et al., 2002) is the most utilized due to its simplicity and versatility. On the other hand, the simplified analytical solutions (Weingarten et al., 1968; Vinson and Sierakowski, 2002; DNV, 2003b) are more practical for design purposes.

In Teófilo et al. (2009) the buckling load and post-buckling behavior of laminated composite tubes subjected to external pressure were evaluated using the FEM and the CLT. Perfect and imperfect tubes with different lay-ups were considered. The FE numerical results were compared with Eq. (25). It was shown that the agreement between the analytical and numerical buckling pressures for long tubes, which is the case of composite riser joints, is satisfactory for preliminary design.

5 OPTIMIZATION MODEL

This section presents the optimization model used for preliminary design of laminated composite risers. Multiple load cases, corresponding to different design scenarios are considered. The input data of the problem includes the loads, material properties, number of plies (n), inner radius (R_i), liner thickness (t_i), Dynamic Amplification Factor, and required safety factors. The objective is to find the best (lower cost) laminate that meets the strength and stability requirements of the composite riser.

The design variables are the thickness (t) and orientation angle (θ) of each ply. Since only symmetric lay-ups are allowed, the vector of design variables is given by

\[ \mathbf{x} = [t_1, t_2, \ldots, t_{n/2}, \theta_1, \theta_2, \ldots, \theta_{n/2}] \]  

These variables are limited by lower and upper bounds:

\[-90^\circ \leq \theta_k \leq 90^\circ, \quad t_{\min} \leq t_k \leq t_{\max}, \quad k = 1, \ldots, n/2 \]  

The formulation aims to minimize the cost of the riser joint, which is assumed proportional to the volume of composite material. Since the section of the tube is assumed constant along the riser length, the objective function is the cross-sectional area of the composite tube, regardless of the liner portion. A normalized value is used to improve the convergence of the optimization algorithm and the objective function (f) is given by

\[ f(\mathbf{x}) = \frac{A - A_{\min}}{A_{\max} - A_{\min}} \]  

where \( A_{\min} \) and \( A_{\max} \) are the minimum and maximum values of the cross-section area:

\[ A_{\max} = \pi \left( R_i^2 + (R_i + t_i)^2 \right); \quad R_{\max} = R_i + t_i + t_{\max} \]

\[ A_{\min} = \pi \left( R_{\min}^2 - (R_i + t_i)^2 \right); \quad R_{\min} = R_i + t_i + t_{\min} \]

Strength and stability constraints are considered in order to obtain a safe riser design. The buckling pressure is computed using Eqs. (24) and (25). For tubes subjected to both internal (p_i) and external pressure (p_e), buckling can occur only when the pressure differential (Δp), defined as
\[
\Delta p = p_i - p_e
\]  

(31)

is negative. Since the tube can be subjected to \(m\) load cases, the Safety Factor related to hoop buckling is computed from

\[
SF_{\text{buck}} = \frac{P_{\text{col}}}{\Delta p_{\text{max}}}, \quad \text{where } \Delta p_{\text{max}} = \text{MAX}(\Delta p_i), \quad l=1,...m
\]  

(32)

In the computer implementation, the stability constraint is written in the normalized form:

\[
-\frac{SF_{\text{buck}}}{SF_{\text{req}}} + 1 \leq 0
\]  

(33)

where \(SF_{\text{req}}\) is the minimum Safety Factor required to avoid hoop buckling, which was defined in Section 4. It is important to note that this constraint is included only if \(\Delta p_{\text{max}} > 0\).

The First Ply Failure (FPF) criterion is used to define laminate failure. This assumption is consistent with the procedure for stress analysis of thin-walled laminate composite tubes described in Section 4.1. It is important to note that the true wall tension \((N_{\text{tw}})\), used in Eq. (19) to the computation of laminate membrane forces, is evaluated from effective tension, internal pressure, and external pressure using Eq. (18). In addition, the effective tension is evaluated using the catenary equations and the Dynamic Amplification Factor, defined in Section 4.

Using the FPF criterion, the laminate Safety Factor \((SF)\) is determined as the smallest \(SF_{kl}\) computed for the laminate plies for all loading cases:

\[
SF = \text{MIN}(SF_{kl}), \quad \text{where } \begin{cases} \{k = 1,...,n \} \\ \{l = 1,...,m \} \end{cases}
\]  

(34)

In this work, the Tsai-Wu and Maximum Stress failure criterion are used to compute the Safety Factor. In the computational implementation, the strength constraint is written in normalized form as

\[
-\frac{SF}{SF_{\text{rec}}} + 1 < 0
\]  

(35)

where \(SF_{\text{rec}}\) is the required Safety Factor, defined in Table 1.

Finally, the optimization model used for preliminary design of composite catenary riser joints is written in compact form as

Find \(x = [t_1 \quad t_2 \quad \ldots \quad t_{n/2} \quad \theta_1 \quad \theta_2 \quad \ldots \quad \theta_{n/2}]\)

That minimizes \(f(x) = \frac{A - A_{\text{min}}}{A_{\text{max}} - A_{\text{min}}}\)

Subjected to:

\[
-\frac{SF_{\text{buck}}}{SF_{\text{req}}} + 1 \leq 0
\]  

(36)

\[
-\frac{SF}{SF_{\text{rec}}} + 1 < 0
\]

\[-90^\circ \leq \theta_k \leq 90^\circ, \quad k=1,\ldots,n/2\]

\(t_{\text{min}} \leq t_k \leq t_{\text{min}}\)
One of the main drawbacks of gradient-based optimization methods is their susceptibility to be attracted by local minima. To overcome this problem, \( N \) initial designs satisfying the bound constraints in \( \theta_k \) and \( t_k \) are randomly generated. These designs are used as starting points for \( N \) independent optimizations carried out sequentially. If the new solution has a smaller objective function value than the current best design, then best design is updated. If a sufficiently high number of different number of starting points is used it is expected that the best design will be the global minimizer of the problem.

6 EXAMPLES

This section presents numerical examples of application of the optimization model described previously to the preliminary design of composite catenary risers. The numerical data used in these examples are presented in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal diameter (m)</td>
<td>0.25</td>
</tr>
<tr>
<td>Maximum number of layers</td>
<td>10</td>
</tr>
<tr>
<td>Liner thickness (m)</td>
<td>0.006</td>
</tr>
<tr>
<td>Minimum and maximum ply thickness (m)</td>
<td>0.001 &lt; ( t_k ) &lt; 0.020</td>
</tr>
<tr>
<td>Fiber orientation angle</td>
<td>-90° &lt; ( \theta ) &lt; 90°</td>
</tr>
<tr>
<td>Required safety factor for buckling</td>
<td>3</td>
</tr>
<tr>
<td>Required safety factor for steel</td>
<td>1.5</td>
</tr>
<tr>
<td>Required safety factor for composite</td>
<td>3.0</td>
</tr>
<tr>
<td>Internal pressure (MPa)</td>
<td>25</td>
</tr>
<tr>
<td>Top angle with vertical</td>
<td>25°</td>
</tr>
<tr>
<td>Floater offset (m)</td>
<td>187.5</td>
</tr>
<tr>
<td>Dynamic Amplification Factor</td>
<td>1.5</td>
</tr>
<tr>
<td>Force coefficient (( \gamma_f ))</td>
<td>1.1</td>
</tr>
<tr>
<td>Thickness of the outer coating 1 (HNBR) (m)</td>
<td>0.002</td>
</tr>
<tr>
<td>Thickness of the outer coating 2 (PP-solid) (m)</td>
<td>0.040</td>
</tr>
<tr>
<td>Specific weight – internal fluid (kN/m³)</td>
<td>7</td>
</tr>
<tr>
<td>Specific weight – water (kN/m³)</td>
<td>10</td>
</tr>
<tr>
<td>Specific weight – HNBR (kN/m³)</td>
<td>9</td>
</tr>
<tr>
<td>Specific weight – PP-solid (kN/m³)</td>
<td>9</td>
</tr>
<tr>
<td>Number of randomly generated initial designs (( N ))</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 2: Numerical data used in the examples.

In all examples two different load cases (A and B) are considered. Load Case A corresponds to the situation of riser with internal fluid (oil) with the floater at the maximum allowable offset in the far direction, while Load Case B corresponds to an empty riser at the maximum allowable offset in the near direction. Therefore, Case A presents a combination of internal pressure and offset in the far direction leading to high tensile stresses in the riser. On the other hand, the combination of an empty riser (\( p_i = 0 \) ) and an offset in the near direction in Case B leads to high compressive stresses in the riser. Therefore, both stress and stability constraints need to be considered in Case B.

6.1 Example 1

In this first example the sea depth is 2500 m, the yield stress of the steel liner is \( f_y = 448 \) MPa, and Tsai-Wu was adopted as the failure criterion for the composite layers. An iteration history (objective function and maximum constraint violation) of the best design is presented in Figure 6. It can be noted that there are large variations on the initial iterations.
until the objective function is close to the minimum and the constraints are satisfied. Further reductions in the objective function are accompanied by constraint violations, but at the end all constraints are satisfied.

Figure 6 – Iteration history of Example 1.

<table>
<thead>
<tr>
<th>Ply</th>
<th>Thickness (mm)</th>
<th>Angle (deg)</th>
<th>SF Case A</th>
<th>SF Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner</td>
<td>6.00</td>
<td>-</td>
<td>1.55</td>
<td>1.50</td>
</tr>
<tr>
<td>1</td>
<td>2.96</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>2</td>
<td>1.03</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>3</td>
<td>3.56</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>4</td>
<td>2.46</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>5</td>
<td>1.20</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>6</td>
<td>1.20</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>7</td>
<td>2.46</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>8</td>
<td>3.56</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>9</td>
<td>1.03</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>10</td>
<td>2.96</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
</tbody>
</table>

Table 3: Optimum layup (Example 1).

<table>
<thead>
<tr>
<th>Ply</th>
<th>Thickness (mm)</th>
<th>Angle (deg)</th>
<th>SF Case A</th>
<th>SF Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner</td>
<td>6.00</td>
<td>-</td>
<td>1.55</td>
<td>1.50</td>
</tr>
<tr>
<td>1</td>
<td>3.58</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>3</td>
<td>4.01</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>4</td>
<td>1.61</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>5</td>
<td>1.01</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>6</td>
<td>1.01</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>7</td>
<td>1.61</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
<tr>
<td>8</td>
<td>4.01</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>0.00</td>
<td>3.94</td>
<td>12.55</td>
</tr>
<tr>
<td>10</td>
<td>3.58</td>
<td>90.00</td>
<td>3.00</td>
<td>11.39</td>
</tr>
</tbody>
</table>

Table 3 presents the laminate data, while Table 4 presents the optimum designs and Safety Factors. It can be noted that both designs are cross-ply laminates, have the same thickness (or objective function), and the same stress safety factor, but Design 1 has greater buckling strength. It can be noted that the stress constraint of the liner is active for Case B.
Table 4: Optimum designs (Example 1)

<table>
<thead>
<tr>
<th></th>
<th>Design 1</th>
<th>Design 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (mm)</td>
<td>28.41</td>
<td>28.41</td>
</tr>
<tr>
<td>Pcol (MPa)</td>
<td>86.54</td>
<td>82.5</td>
</tr>
<tr>
<td>$SF$</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>$SF_{buck}$</td>
<td>3.15</td>
<td>3.00</td>
</tr>
</tbody>
</table>

6.2 Example 2

The numerical data used in this example are the same of the previous one, but the yield stress of the steel liner was increased to $f_y = 551$ MPa. In this case only one global optimum design, presented in Table 5, was found. The total thickness of the laminate is 27.25 mm, the collapse (buckling) pressure is 82.5 MPa, and both stress and stability safety factors are equal to 3.0. It is interesting to note the stress constraints are active for Case A, while the stability constraint is active for Case B. Furthermore, the comparison of this design of with designs of the previous example shows that the use of a liner with higher strength leads to a laminate with smaller thickness.

<table>
<thead>
<tr>
<th>Ply</th>
<th>Thickness (mm)</th>
<th>Angle (deg)</th>
<th>SF Case A</th>
<th>SF Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner</td>
<td>6.00</td>
<td>0.00</td>
<td>1.88</td>
<td>1.77</td>
</tr>
<tr>
<td>1</td>
<td>4.60</td>
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<td>3.00</td>
<td>10.69</td>
</tr>
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<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
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<td>1.00</td>
<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
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<td>4</td>
<td>1.00</td>
<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
</tr>
<tr>
<td>5</td>
<td>2.99</td>
<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
</tr>
<tr>
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<td>2.99</td>
<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
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<tr>
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<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
</tr>
<tr>
<td>8</td>
<td>1.00</td>
<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
</tr>
<tr>
<td>9</td>
<td>1.02</td>
<td>0.00</td>
<td>3.80</td>
<td>11.96</td>
</tr>
<tr>
<td>10</td>
<td>4.60</td>
<td>90.00</td>
<td>3.00</td>
<td>10.69</td>
</tr>
</tbody>
</table>

Table 5: Optimum layup (Example 2).

6.3 Example 3

The numerical data used in this example are the same of the Example 1, but the sea depth was increased to 3000 m. Two different designs, both with cross-ply layups, as presented in Table 6, were found. The thickness, collapse pressure, and safety factors of both laminates are presented in Table 7. As expected, greater sea depths lead to higher external pressure and greater axial forces, requiring a large laminate thickness in comparison to Example 1.
Optimum design 1

<table>
<thead>
<tr>
<th>Ply</th>
<th>Thickness (mm)</th>
<th>Angle (deg)</th>
<th>SF Case A</th>
<th>SF Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner</td>
<td>6.00</td>
<td>-</td>
<td>1.64</td>
<td>1.50</td>
</tr>
<tr>
<td>1</td>
<td>5.90</td>
<td>0.00</td>
<td>4.67</td>
<td>12.84</td>
</tr>
<tr>
<td>2</td>
<td>6.54</td>
<td>90.00</td>
<td>3.00</td>
<td>12.08</td>
</tr>
<tr>
<td>3</td>
<td>2.13</td>
<td>0.00</td>
<td>4.67</td>
<td>12.84</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>90.00</td>
<td>3.00</td>
<td>12.08</td>
</tr>
<tr>
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<td>90.00</td>
<td>3.00</td>
<td>12.08</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>90.00</td>
<td>3.00</td>
<td>12.08</td>
</tr>
<tr>
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<td>1.00</td>
<td>90.00</td>
<td>3.00</td>
<td>12.08</td>
</tr>
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<td>0.00</td>
<td>4.67</td>
<td>12.84</td>
</tr>
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<td>6.54</td>
<td>90.00</td>
<td>3.00</td>
<td>12.08</td>
</tr>
<tr>
<td>10</td>
<td>5.90</td>
<td>0.00</td>
<td>4.67</td>
<td>12.84</td>
</tr>
</tbody>
</table>

Optimum design 2

<table>
<thead>
<tr>
<th>Ply</th>
<th>Thickness (mm)</th>
<th>Angle (deg)</th>
<th>SF Case A</th>
<th>SF Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner</td>
<td>6.00</td>
<td>-</td>
<td>1.64</td>
<td>1.50</td>
</tr>
<tr>
<td>1</td>
<td>1.03</td>
<td>0.00</td>
<td>4.67</td>
<td>12.84</td>
</tr>
<tr>
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<td>90.00</td>
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<td>1.03</td>
<td>0.00</td>
<td>4.67</td>
<td>12.84</td>
</tr>
</tbody>
</table>

Table 6: Optimum layups (Example 3).

It can be noted that both laminates have the same total thickness, but different stacking sequences. This occurs because the stress safety factor of the laminate depends on the total axial (0°) and hoop (90°) thickness, but not on the stacking sequence, since bending was not considered in the stress analysis. On the other hand, Design 2 has a high buckling pressure, which is a desired feature.

<table>
<thead>
<tr>
<th>Design 1</th>
<th>Design 2</th>
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</thead>
<tbody>
<tr>
<td>Thickness (mm)</td>
<td>39.14</td>
</tr>
<tr>
<td>$P_{col}$ (MPa)</td>
<td>100.2</td>
</tr>
<tr>
<td>$SF$</td>
<td>3.00</td>
</tr>
<tr>
<td>$SF_{buck}$</td>
<td>3.04</td>
</tr>
</tbody>
</table>

Table 7: Optimum designs (Example 3).

6.4 Example 4

The numerical data used in this example are the same as those of Example 1, but the Maximum Stress Criterion instead of the Tsai-Wu Criterion. Once again the two optimum laminates were found, both with cross-ply layups (Table 8).
The thickness, collapse pressure, and safety factors of both laminates are presented in Table 9. It can be noted that both laminated have the same thickness, but laminate 2 has a greater buckling pressure. Finally, the comparison with Example 1 shows that the use of Tsai-Wu Criterion leads to more economic riser designs than the use of the Maximum Stress Criterion.

<table>
<thead>
<tr>
<th>Ply</th>
<th>Thickness (mm)</th>
<th>Angle (deg)</th>
<th>SF Case A</th>
<th>SF Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner</td>
<td>6.00</td>
<td>-</td>
<td>1.61</td>
<td>1.50</td>
</tr>
<tr>
<td>1</td>
<td>1.16</td>
<td>89.99</td>
<td>3.00</td>
<td>6.20</td>
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<td>5.89</td>
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<td>3.95</td>
<td>12.33</td>
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<tr>
<td>3</td>
<td>2.15</td>
<td>89.98</td>
<td>3.00</td>
<td>6.20</td>
</tr>
<tr>
<td>4</td>
<td>4.04</td>
<td>90.00</td>
<td>3.00</td>
<td>6.20</td>
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<td>-0.01</td>
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<td>1.16</td>
<td>89.99</td>
<td>3.00</td>
<td>6.20</td>
</tr>
</tbody>
</table>

Table 8: Optimum layups (Example 4).

<table>
<thead>
<tr>
<th>Ply</th>
<th>Thickness (mm)</th>
<th>Angle (deg)</th>
<th>SF Case A</th>
<th>SF Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liner</td>
<td>6.00</td>
<td>-</td>
<td>1.61</td>
<td>1.50</td>
</tr>
<tr>
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<td>90.00</td>
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<td>90.00</td>
<td>3.00</td>
<td>6.20</td>
</tr>
</tbody>
</table>

Table 9: Optimum designs (Example 4).

7 CONCLUSIONS

This paper presented a methodology to the design of composite catenary risers for deepwater applications. The design of steel catenary risers is already very complex, but the design of composite risers is even more complex due to the use of the global-local approach for structural analysis and the large number of parameters to define the lamination scheme of the composite joint.

Due to the complexity of the problem it is not feasible yet to apply an optimization technique to the detailed design of composite risers. Therefore, optimization techniques were applied in this work to obtain a preliminary design of composite riser joints. The design variables are thickness and orientation of each layer and the objective is to minimize the volume of composite material. The axial force is estimated using the catenary equations and the stresses and buckling loads are evaluated using the Classical Lamination Theory (CLT). Strength and stability constraints are included and different load cases are considered in the optimization model.

The results showed that the proposed methodology works well and is able to find good
preliminary designs even using random generated initial designs. Solutions were found even starting from infeasible initial points in the design space. In most examples, more than one global optimum design with the same thickness and stress safety factor. In this case, it is suggested to choose the laminate with greater buckling pressure. As expected, the required composite volume increases with the water depth. Finally, the use of Tsai-Wu Criterion leads to more economic designs than the Maximum Stress Criterion.

REFERENCES


