Computing the Yield from an Infinite Reservoir

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Abstract. The present paper presents a procedure for preliminary estimations of the efficiency (η_M) of a reservoir having infinite capacity. Monte Carlo Method was used to find the yields under different conditions, such as: coefficient of variation of annual inflows (CV) ranging from 0.5 to 1.6 and dimensionless evaporation factor (f_E) ranging from 0.05 to 2.0. Assuming infinite storage, the reservoir was simulated on its steady state (simulation horizon equal to 2,000 years) and the annual reliability (G) was assumed equal to 90% . The basic assumptions used to develop the model were: time is discrete and time step is a year; reservoir volume is discrete; serial correlation of annual inflows is zero; all inflows occur in a wet season and all output in a dry season and inflows come from a Gamma II distribution. The equation for reservoir efficiency has the following form:

$$\eta_M = 0.99 \exp [-f_E / (1.5031 - 1.7104CV + 0.8555CV^2 - 0.1528CV^3)$$

The main objective is to provide, quickly and with a certain accuracy, a tool for estimate the efficiency of an infinite reservoir.

1. Statement of the Problem

The use of equations to estimate reservoir yield has been studied by engineers and researchers all over the world. Some procedures and expressions to make preliminary evaluation on reservoir studies are presented in McMahon and Mein (1978).

The major disadvantage of these simplified methods comes exactly from their most advantage: the simplicity. To make the mathematical computations easier various simplifications must be embodied on them. Hence, no one can expect that the same procedure can give results as accurate for two very dissimilar hydrological regimen, such as intermittent rivers in Northeast Brazil and perennial rivers in temperate Northeast USA. However, it is possible to develop a parametrical procedure and a mathematical process to find a general expression suitable to different regions. Distinct equations can be found just changing input parameters.

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2. Basic Concepts

The storage process can be represented by the equation:

$$M = \Phi\ (CV,\ Z_0\ , H, G, K, \alpha, E_v) \quad (1)$$

where M is the programmed annual yield, CV is the coefficient of variation of annual inflows, Z_0 is the reservoir initial storage, H is the time horizon adopted in simulation, G is the annual system reliability, K is the reservoir capacity, α is the reservoir shape factor and E_v is the mean evaporation depth during the dry season.

This equation can be reduced to its dimensionless form using the parameters defined by Campos (1987):

$$f_M = \Phi\ (CV, G, f_K, f_E, Z_t, H) \quad (2)$$

where: f_K is the dimensionless capacity factor, f_E is the dimensionless evaporation factor and f_M is the dimensionless factor of release, given by:

$$f_K = K/\mu \quad (3)$$

$$f_E = (3\alpha^{1/3}E_V) / \mu^{1/3} \quad (4)$$

$$f_M = M/\mu \quad (5)$$

where μ is the mean annual inflow.

A simple procedure to wash out the initial storage effect on computations using Monte Carlo technique is to use a very high value for time horizon (H) (Studart and Campos, 1999). In this paper it was assumed H=2,000 years, G=90% and K =infinite (the reservoir can empty but never spills). So, the equation (2) takes the form:

$$f_M = \Phi\ (CV, f_E) \quad (6)$$

If the reliability is assumed to be 90% (G_{90}), it means that failures will take place in 10% of the years. Better saying, in 90% of the years the release (R) will be equal to the programmed yield (M); in failure years R will be less than M. So, the actual release (R_a) for all *n* years is given by:

$$R_a = \Sigma\ R_i / n \quad (7)$$

A good approximation to R_a is to make:

$$R_a = 0.95\ M \tag{8}$$

If the reservoir efficiency (η_M) is defined by:

$$\eta_M = R_a/\mu \tag{9}$$

it can be said that the reservoir efficiency (η_M) for 90% of reliability (G_{90}), in terms of the dimensionless factor of release (f_M) is given by:

$$\eta_M = 0.95\ f_M \tag{10}$$

3. Methodology

3.1. Range of input parameters

To define the range for input parameters - CV and f_E - it was analyzed a collection of hydrologic studies developed by the Grupo Executivo de Irrigação para o Desenvolvimento Agrícola (GEIDA) (1971). These studies covered most of Brazil's Northeast dams existent at that time. As a result we got a range wide enough to permit application of the procedures to other areas around the world with similar conditions. The range, for CV and f_E is:

- f_E – 0.05 to 2.0
- CV – 0.5 to 1.6

3.2. Model development

The basic assumptions used to develop the model is explained by Campos (1987):

1. Time is discrete and time step is a year;
2. Reservoir volume is discrete;
3. Serial correlation of annual inflows is zero;
4. All inflows occurs in a wet season and all output in a dry season;
5. Inflows come from Gamma II distribution

3.3. Synthetic series generation

The intermittent rivers in Brazilian Northeast present, as their characteristic, a hydrological regimen with two well-defined seasons – a wet season where all flows occurs and a long dry season with no flows. Those characteristics make the annual discharge serial independent; so, they can be obtained from random numbers generation following a probability density function. According to Campos (1996), both Log-normal and Gamma can be used in those cases; the Gamma Function was selected here.

3.4. Reservoir simulation

The reservoir water budget can be represented by equation 11 and 12, as follows:

$$Z_{t+1} = Z_t + I_t - M - (1/2).(A_{t+1} + A_t).E - S_t \qquad (11)$$

and:

$$S_t = \max(Z_t + I_t - M.(1/2).(A_{t+1} + A_t).E_v - K; 0) \qquad (12)$$

where Z_{t+1} and Z_t – storage at the beginning of the $(t+1)^{th}$ and t^{th} years, respectfully; I_t – inflow into the reservoir during the t^{th} year; M – release from the reservoir during year the t^{th} (M is assumed constant from year to year); A_{t+1} and A_t – lake area at the beginning of the $(t+1)^{th}$ and t^{th} years; E_v - mean evaporation depth during the dry season (E_v is assumed constant from year to year); K - reservoir capacity and S_t - spill from the reservoir during the t^{th} year

The water budget equation was solved using a numerical integration process. It is assumed a two-season process with inflow concentrated in the wet season and outflow concentrated in the dry season - the "mutually exclusive" model - given by Moran (1954). In other words, the assumption is based on transferring small amounts of inputs that occur in the dry season to the wet season, and a small demand of the wet season to the dry season.

4. Equation Development

To find the dimensionless factor of release (f_M) and, consequently, a mathematical expression for the reservoir efficiency (η_M), 2,000 traces were generated from a Gamma II population having CV varying from 0.5 to 1.6. Each one of those 12 synthetic series of inflows were then routed through the reservoir, and for each value of f_E, a different value of f_M - and η_M - was determined.

For each value of CV, the dimensionless factor of release f_M was related by least square analysis to the dimensionless factor of evaporation f_E. The 12 curves f_M x f_E are showed in Figure 1. The equations of these curves are in Table 1.

The equations have the general form :

$$f_M = a_1.\exp[-(f_E/b_1)] \qquad (13)$$

where the constants a_1 and b_1 are function of CV.

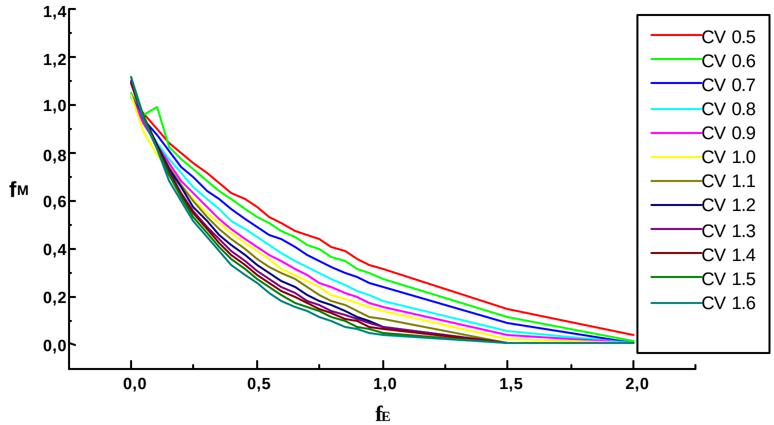

Figure 1. Relation of f_M and f_E for CV's ranging from 0.5 to 1.6 and f_E ranging from 0.05 to 2.0

Table 1. Equations for the curves f_M x f_E

CV	Equations
0.5	f_M = 1.02522exp (- f_E / 0.83932)
0.6	f_M = 1.02880exp (- f_E / 0.75482)
0.7	f_M = 1.02026exp (- f_E / 0.68061)
0.8	f_M = 1.01498exp (- f_E / 0.59574)
0.9	f_M = 1.01681exp (- f_E / 0.53752)
1.0	f_M = 1.00113exp (- f_E / 0.51065)
1.1	f_M = 1.06412exp (- f_E / 0.44862)
1.2	f_M = 1.08000exp (- f_E / 0.41159)
1.3	f_M = 1.07878exp (- f_E / 0.38899)
1.4	f_M = 1.08746exp (- f_E / 0.37141)
1.5	f_M = 1.10410exp (- f_E / 0.34858)
1.6	f_M = 1.10927exp (- f_E / 0.32757)

The values of f_M found by simulation and equation are very close; the figures 2, 3 and 4 make the comparison for CV=0.9, 1.0 and 1.1.

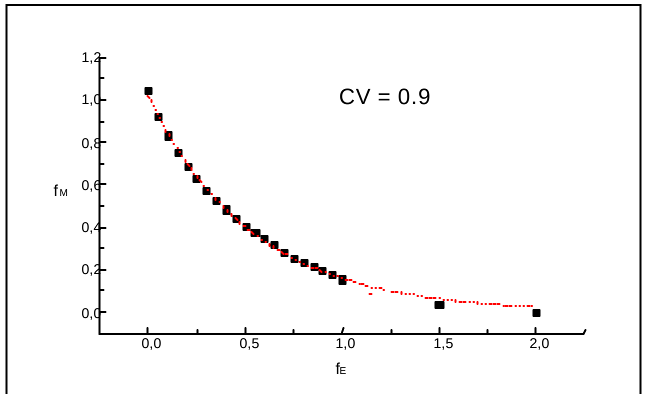

Figure 2. Values of f_M found by equation (*line*) and simulation (*dots*) for CV=0.9

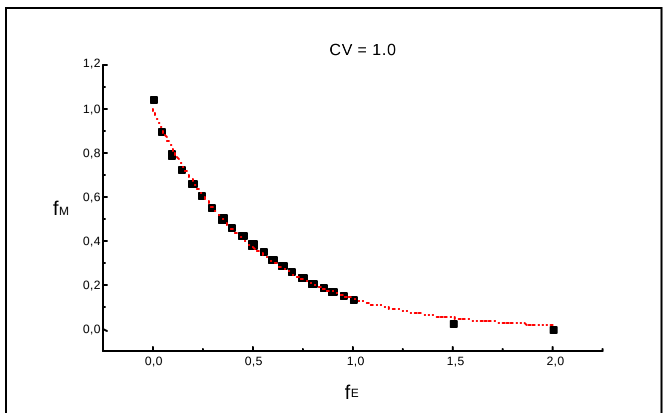

Figure 3. Values of f_M found by equation (*line*) and simulation (*dots*) for CV=1.0

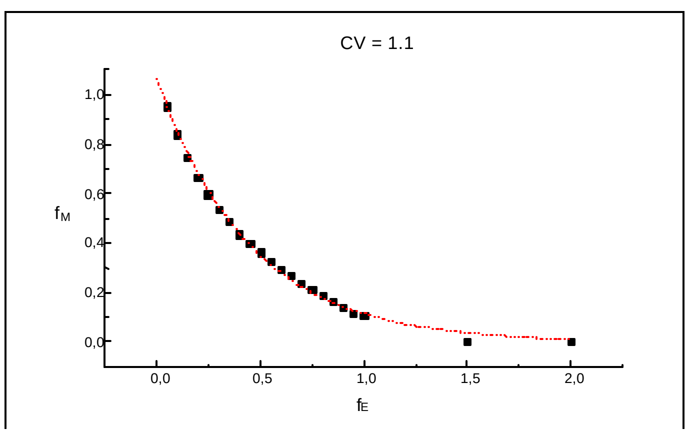

Figure 4. Values of f_M found by equation (*line*) and simulation (*dots*) for CV=1.1

As can be saw in Table 1 the values assumed by the constant a_1 are very close to each other (the variation was 1.02522 to 1.10927). The constant b_1, however, had a faster response to CV (0.83932 to 0.32757). In other words, a_1 is less sensitive to CV variations than b_1. Calculating the mean value of a_1 and the regression equation between CV and b_1, one can find a general equation for f_M. Once f_M is related to η_M by a factor of 0.95, the efficiency of an infinite reservoir is given by:

$$\eta_M = 0.99 \exp [-f_E / (1.5031 - 1.7104CV + 0.8555CV^2 - 0.1528CV^3) \quad (14)$$

6. Conclusions

This paper described a reasonable quick procedure for preliminary estimations of reservoirs' efficiency (η_M). Assuming infinite storage and reliability of 90%, the authors obtained a general equation for η_M on the steady state of the storage process. Only two input parameters are necessary: the coefficient of variation of annual inflows (CV) and the dimensionless factor of evaporation (f_E). Analyzing values for both simulation and equation it seemed that the procedure is very accurate to semiarid conditions (CV=0.5 to 1.6 and f_E=0.05 to 2.0). The equation was not yet tested for values outside this range.

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