A QR FACTORIZATION BASED ALGORITHM FOR PILOT ASSISTED CHANNEL ESTIMATION IN OFDM SYSTEMS

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ABSTRACT

This paper deals with the pilot assisted time-varying channel estimation in *orthogonal frequency division multiplexing* (OFDM) systems. Since the pilot sub-carrier correlations are rank deficient, an estimation scheme can be implemented by discarding the noise subspace components and filtering the subspace components. The subspace can be estimated by some *subspace tracking* (ST) algorithm. We propose to implement the subspace components filtering via a QR factorization based algorithm, which is suitable for operating in conjunction with the ST algorithm. The performance of the proposed schemes is verified by simulation in computer.

***Index Terms*—** Frequency division multiplexing, Time-varying channels, Matrix decomposition.

1. INTRODUCTION

In pilot assisted estimation of *orthogonal frequency division multiplexing* (OFDM) channels, the simple *least-squares* (LS) approach leads to very inaccurate estimates for high *signal-to noise ratio* (SNR) [1]. In order to obtain less noisy estimates, the LS processing output can be additionally filtered [2]. Since the pilot sub-carriers correlations are rank deficient, we benefit from a projection in the signal subspace [3]. Further, the noise in the projection components can be attenuated via Wiener filtering. A *subspace tracking* (ST) algorithm can perform the signal subspace estimation. In this work, we used the LORAF3 algorithm [4]. The subspace components are estimated by an adaptive filtering based on QR factorization. The QR approach is suitable for a simultaneous operation of both ST algorithm and the adaptive filter. The performance of the proposed algorithms was verified by computer simulation. The proposed algorithm is also compared with the low-pass filtering approach [5]. The rest of this is organized as follows. In section 2, we present the filtering structure on which the algorithms are based. In section 3, the proposed algorithms are derived. The simulation results are shown in section 4. Finally, we draw the conclusions in section 5.

2. FILTERING STRUCTURE

In this section we derive the filtering structure based on subspace decomposition and filtering over the subspace components.

The received signal $x[m,k]$ and transmitted symbol $a[m,k]$ at the m-th OFDM symbol and k-th subcarrier can be related as

$$x[m,k] = H[m,k]a[m,k] + u[m,k] + w[m,k],$$

where $H[m,k]$ is the subcarrier complex attenuation, and $u[m,k]$ and $w[m,k]$ are the *inter-carrier interference* (ICI) and noise components, respectively. We consider that the symbols $a[m,k]$ are uncorrelated for different m's and k's. The noise $w[m,k]$ is supposed i.i.d. and independent of the remaining signals. Supposing that $a[m,k]$ at the pilot positions are selected from a PSK constellation, the LS estimate of $H[m,k]$ can be found simply back-rotating $x[m,k]$, which results in

$$\tilde{H}[m,k] = x[m,k]a^*[m,k] = H[m,k] + z[m,k]$$

where $z[m,k] = (u[m,k] + w[m,k])a^*[m,k]$. Due to the uncorrelated assumption of $a[m,k]$, we can write

$$\mathbb{E}\{z^*[m_1,k_1]z[m_2,k_2]\} = 0, \qquad \text{for } m_1 \neq m_2 \text{ or } k_1 \neq k_2,$$
$$\mathbb{E}\{H^*[m_1,k_1]z[m_2,k_2]\} = 0, \qquad \text{for any } m_1, m_2, k_1, k_2.$$

This simplifies significantly the design of a second-order estimator for $H[m,k]$ (see [2]). We consider the pilot subcarriers are arranged in *grid*. The pilot subcarriers are allocated at positions $m = nM_t$ and $k = lM_f$, for $n \in \mathbb{N}$ and $l = 0, \dots, N_p - 1$, where N_p is the number of pilot subcarriers per OFDM symbol. Let $\tilde{\mathbf{H}}[n]$, $\mathbf{H}[n]$ and $\mathbf{z}[n]$ be column vectors containing respectively $\tilde{H}[nM_t, lM_f]$, $H[nM_t, lM_f]$ and $z[nM_t, lM_f]$, for $l = 0, \dots, N_p - 1$. Then we can state

$$\mathbf{R}_{\tilde{H}} = \mathbb{E}\{\tilde{\mathbf{H}}[n]\tilde{\mathbf{H}}^H[n]\} = \mathbf{R}_H + \rho\mathbf{I},$$

where $\mathbb{E}\{\cdot\}$ is the expectation operator, $\mathbf{R}_H = \mathbb{E}\{\mathbf{H}[n]\mathbf{H}^H[n]\}$ and ρ is the variance of $z[nM_t, lM_f]$. Let $\mathbf{U}$ be the unitary matrix associated to the $K \leq N_p$ largest eigenvalues of $\mathbf{R}_H$, such that the signal of interest $\mathbf{H}[n]$ lies in the space spanned by the columns of $\mathbf{U}$. The projection of $\tilde{\mathbf{H}}[n]$ over this space has a component vector

$$\tilde{\mathbf{d}}[n] = \mathbf{U}^H\tilde{\mathbf{H}}[n] = \mathbf{d}[n] + \eta[n],$$

where $\mathbf{d}[n] = \mathbf{U}^H\mathbf{H}[n]$ and $\eta[n] = \mathbf{U}^H\mathbf{z}[n]$. Let $\tilde{d}[n,l]$, $d[n,l]$ and $\eta[n,l]$ denote the l-th element of $\tilde{\mathbf{d}}[n]$, $\mathbf{d}[n]$ and $\eta[n]$, respectively. The desired term $d[n]$ can be estimated via a MMSE approach. Obviously we could choose any other filtering criterion for estimating $d[n,l]$ from $\hat{d}[n,l]$. A straightforward computation leads to the Wiener filter of coefficients

$$\mathbf{c}[l] = \mathbf{R}_{\tilde{d}}^{-1}[l]\mathbf{r}_{\tilde{d}d}[l]. \tag{1}$$

where $\tilde{\mathbf{d}}[n;l] = (\tilde{d}[n,l], \dots, \tilde{d}[n-M+1,l])^T$ is the vector containing the last M realizations of $\tilde{d}[n,l]$, $\mathbf{R}_{\tilde{d}}[l] = \mathbb{E}\{\tilde{\mathbf{d}}[n;l]\tilde{\mathbf{d}}^H[n;l]\}$ and $\mathbf{r}_{\tilde{d}d}[l] = \mathbb{E}\{\tilde{\mathbf{d}}[n;l]d[n;l]\}$. Then we obtain the Wiener estimate of $d[n;l]$ as

$$\hat{d}[n,l] = \mathbf{c}^H[l]\tilde{\mathbf{d}}[n;l].$$

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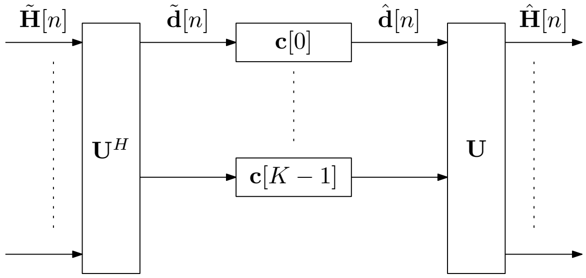

Fig. 1. Filter design for estimation of pilot subcarriers.

And the pilot sub-carriers are finally recovered as

$$\hat{\mathbf{H}}[n] = \mathbf{U}\hat{\mathbf{d}}[n],$$

where $\hat{\mathbf{d}}[n]$ is the column vector whose l-th element is $\hat{d}[n,l]$.

The resulting filtering structure is illustrated in Fig. 1. The remaining sub-carriers can be estimated by some interpolation technique [6]. An improvement in the accuracy of the LS estimates results in more accurate sub-carrier estimates in the interpolation outputs. In the next section we propose an adaptive scheme for estimating the pilot sub-carriers.

3. PROPOSED ALGORITHMS

In order to estimate the pilot subcarriers, $\mathbf{U}$ and $\mathbf{c}[l]$ should be known first. The matrix $\mathbf{U}$ can be estimated by some ST algorithm. For example, we could employ the LORAF algorithm, which is summarized in Table 1. Obviously, the ST algorithms require a value $K_{\max}$ for the subspace dimension, which should satisfies $K < K_{\max}$ in order to operate with no losses. The choice of $K_{\max}$ depends on the channel environment and the supported computational complexity. The performance can be improved if the remaining $K_{\max} - K$ components are discarded. For this, we should estimate K. If σ_H^2 is the subcarrier variance, we can write

$$P_{\tilde{H}} = \mathbb{E}\{\tilde{\mathbf{H}}^H[n]\tilde{\mathbf{H}}[n]\} = N_p\sigma_H^2 + N_p\rho, \qquad P_{\tilde{d}} = \mathbb{E}\{\tilde{\mathbf{d}}^H[n]\tilde{\mathbf{d}}[n]\} = N_p\sigma_H^2 + K_{\max}\rho,$$

which provides

$$\rho = \frac{P_{\tilde{H}} - P_{\tilde{d}}}{N_p - K_{\max}}. \qquad (2)$$

Inserting in Eq. (2) the recursive estimates for $P_{\tilde{H}}$ and $P_{\tilde{d}}$:

$$P_{\tilde{H}}[n] = \alpha P_{\tilde{H}}[n-1] + (1-\alpha)\tilde{\mathbf{H}}^H[n]\tilde{\mathbf{H}}[n],$$
$$P_{\tilde{d}}[n] = \alpha P_{\tilde{d}}[n-1] + (1-\alpha)\tilde{\mathbf{d}}^H[n]\tilde{\mathbf{d}}[n],$$

we obtain the following estimate for ρ:

$$\hat{\rho}[n] = \alpha \cdot \hat{\rho}[n-1] + (1-\alpha)\cdot(\tilde{\mathbf{H}}^H[n]\tilde{\mathbf{H}}[n] - \tilde{\mathbf{d}}^H[n]\tilde{\mathbf{d}}[n])/(N_p - K_{\max}), \qquad (3)$$

where the factor α satisfies $0 < \alpha < 1$. Since $p[l] = \mathbb{E}\{|\tilde{d}[n,l]|^2\} > \rho$ for $l < K$, and $p[l] = \rho$ for $l \geq K$, we can find K by selecting the largest l such that $\mathbb{E}\{|\tilde{d}[n,l]|^2\} > \rho$. For each l, we estimate $p[l]$ as

$$p[n,l] = \alpha \cdot p[n-1,l] + (1-\alpha)\cdot|\tilde{d}[n,l]|^2.$$

<u>Initialization:</u>
$\mathbf{U}[0] = \binom{\mathbf{I}}{\mathbf{0}}$; $\quad 0 < \alpha < 1$;
<u>For each n:</u>
$\mathbf{h}[n] = \mathbf{U}^H[n-1]\tilde{\mathbf{H}}[n]$
$\tilde{\mathbf{H}}_\perp = \tilde{\mathbf{H}}[n] - \mathbf{U}[n-1]\mathbf{h}[n]$
$Z[n] = \tilde{\mathbf{H}}_\perp^H[n]\tilde{\mathbf{H}}_\perp[n]$
$\bar{\tilde{\mathbf{H}}}_\perp[n] = Z^{-1/2}[n]\tilde{\mathbf{H}}_\perp[n]$
$\begin{pmatrix}\mathbf{R}[n]\\ 0\cdots 0\end{pmatrix} = \mathbf{G}[n]\begin{pmatrix}\alpha\mathbf{R}[n-1] + (1-\alpha)\mathbf{h}[n]\mathbf{h}^H[n]\\ (1-\alpha)Z^{1/2}[n]\mathbf{h}^H[n]\end{pmatrix}$
$(\mathbf{U}[n] \mid \star) = (\mathbf{U}[n-1] \mid \bar{\tilde{\mathbf{H}}}_\perp[n])\mathbf{G}^H[n]$

Table 1. The LORAF3 algorithm.

<u>Initialization:</u>
$K_{\max} \geq K$; $\quad \hat{\rho}[n] = 0$; $\quad p[n,l] = 0$; $\quad 0 < \alpha < 1$; $\quad \beta > 1$;
<u>For each n:</u>
$\hat{\rho}[n] = \alpha\cdot\hat{\rho}[n-1] + (1-\alpha)\cdot(\tilde{\mathbf{H}}^H[n]\tilde{\mathbf{H}}[n] - \tilde{\mathbf{d}}^H[n]\tilde{\mathbf{d}}[n])/(N_p - K_{\max})$
$p[n,l] = \alpha\cdot p[n-1,l] + (1-\alpha)\cdot|\tilde{d}[n,l]|^2$
$\hat{K}[n] = \#\{p[n,l]; p[n,l] > \beta\cdot\hat{\rho}[n]\}$

Table 2. Estimation of ρ and K.

And finally we have the estimate

$$\hat{K}[n] = \#\{p[n,l]; p[n,l] > \beta\cdot\hat{\rho}[n]\},$$

where the symbol $\#\{\cdot\}$ stands for the cardinality of the set in the argument. The factor $\beta > 1$ was inserted in order to avoid the wrong selection of l satisfying $p[n,l] > \hat{\rho}[n]$ for $l \geq K$, which can happen due to estimation errors.

The resulting algorithm is summarized in Table 2. In what follows, we derive an adaptive estimate of $\mathbf{c}[l]$. The estimate $\hat{\rho}[n]$ obtained above will be employed in the recursive formula of $\mathbf{c}[l]$.

Denoting $\mathbf{r}_{\tilde{d}\tilde{d}}[l] = \mathbb{E}\{\tilde{\mathbf{d}}[n;l]\tilde{d}[n,l]\}$ and $\mathbf{e}_1 = (1,0,\ldots,0)^T$, we can write

$$\mathbf{r}_{\tilde{d}d}[l] = \mathbf{r}_{\tilde{d}\tilde{d}}[l] - \rho\mathbf{e}_1, \qquad \mathbf{R}_{\tilde{d}}^{-1}[l]\mathbf{r}_{\tilde{d}d}[l] = \mathbf{e}_1,$$

which, together with Eq. 1, results in

$$\mathbf{c}[l] = \mathbf{e}_1 - \rho\mathbf{R}_{\tilde{d}}^{-1}[l]\mathbf{e}_1.$$

We can estimate $\mathbf{R}_{\tilde{d}}[l]$ as

$$\mathbf{R}_{\tilde{d}}[n;l] = \alpha\mathbf{R}_{\tilde{d}}[n-1;l] + (1-\alpha)\tilde{\mathbf{d}}[n;l]\tilde{\mathbf{d}}^H[n;l]. \qquad (4)$$

From the Woodbury identity, the inverse $\mathbf{P}[n;l] = \mathbf{R}_{\tilde{d}}^{-1}[n;l]$ can be recursively computed as

$$\mathbf{P}[n;l] = \alpha^{-1}\mathbf{P}[n-1;l] - \alpha^{-1}\mathbf{k}[n;l]\tilde{\mathbf{d}}^H[n;l]\mathbf{P}[n-1;l], \qquad (5)$$

where

$$\mathbf{k}[n;l] = \frac{\mathbf{P}[n-1;l]\tilde{\mathbf{d}}[n;l]}{\alpha(1-\alpha)^{-1} + \tilde{\mathbf{d}}^H[n;l]\mathbf{P}[n-1;l]\tilde{\mathbf{d}}[n;l]}. \qquad (6)$$

If we define $\mathbf{p}[n;l]$ as the first column of $\mathbf{P}[n;l]$, we obtain the recursive estimate

$$\mathbf{c}[n;l] = \mathbf{e}_1 - \hat{\rho}[n]\mathbf{p}[n;l],$$

and, consequently,

$$\hat{d}[n;l] = \tilde{d}[n;l] - \hat{\rho}[n](\mathbf{p}^H[n;l]\tilde{\mathbf{d}}[n;l]),$$

where $\hat{\rho}[n]$ is the estimate of ρ given in Eq. 3.

The obtained algorithm is given in Table 3.

From Eq. 5, we have the following recursive formula

$$\mathbf{p}[n;l] = \alpha^{-1}\mathbf{p}[n-1;l] - \alpha^{-1}\mathbf{k}[n;l](\tilde{\mathbf{d}}^H[n;l]\mathbf{p}[n-1;l]). \quad (7)$$

The updating of $\mathbf{p}[n;l]$ depends on $\mathbf{k}[n;l]$, whose computation requires a complexity of $O(M^2)$. Fortunately, there exist fast algorithms that perform a recursive computation of $\mathbf{k}[n;l]$. In this case, the estimation of $\mathbf{d}[n]$ would require a computational complexity of $O(MK_{\max})$.

Since $\mathbf{U}$ and $\mathbf{c}[l]$ are estimated simultaneously, the estimation errors of $\mathbf{U}[n]$ can lead to instabilities in the obtained algorithm. Indeed, an explosive divergence was observed in our computational simulations. In order to avoid this, we design an algorithm based on QR factorization [7], which is numerically stable and can operate over changes in the correlations of $\tilde{d}[n,l]$.

Let $\mathbf{R}_{\tilde{d}}^{1/2}[n;l]$ be the root square of $\mathbf{R}_{\tilde{d}}[n;l]$, such that

$$\mathbf{R}_{\tilde{d}}[n;l] = \mathbf{R}_{\tilde{d}}^{1/2}[n;l]\mathbf{R}_{\tilde{d}}^{H/2}[n;l],$$

where $\mathbf{R}_{\tilde{d}}^{H/2}[n;l]$ is upper triangular. If we organize the terms in Eq. (4) as

$$\mathbf{G}[n;l] = \begin{pmatrix} \alpha\mathbf{R}_{\tilde{d}}[n;l] & (1-\alpha)^{1/2}\tilde{\mathbf{d}}[n;l] \\ (1-\alpha)^{1/2}\tilde{\mathbf{d}}^H[n;l] & 1 \end{pmatrix}.$$

we can write

$$\mathbf{G}[n;l] = \mathbf{A}[n;l]\mathbf{A}^H[n;l],$$

where

$$\mathbf{A}[n;l] = \begin{pmatrix} \alpha^{1/2}\mathbf{R}_{\tilde{d}}^{1/2}[n-1;l] & (1-\alpha)^{1/2}\tilde{\mathbf{d}}[n;l] \\ \mathbf{0}^H & 1 \end{pmatrix}$$

Applying a sequence of Givens rotations to $\mathbf{A}[n;l]$, we can obtain the factorization

$$\mathbf{A}[n;l]\Theta[n;l] = \mathbf{B}[n;l] = \begin{pmatrix} \mathbf{B}_{11}[n;l] & \mathbf{0} \\ \mathbf{b}_{21}^H[n;l] & b_{22}[n;l] \end{pmatrix},$$

where $\mathbf{B}_{11}[n;l]$ is upper triangular. Since $\Theta[n;l]$ is unitary, we have

$$\mathbf{A}[n;l]\mathbf{A}^H[n;l] = \mathbf{B}[n;l]\mathbf{B}^H[n;l],$$

whose expansion provides the relations

$$\begin{aligned} \mathbf{B}_{11}[n;l] &= \mathbf{R}_{\tilde{d}}^{1/2}[n;l] \\ \mathbf{b}_{21}[n;l] &= (1-\alpha)^{1/2}\mathbf{R}_{\tilde{d}}^{-1/2}[n;l]\tilde{\mathbf{d}}[n;l] \end{aligned} \quad (8)$$

From $\mathbf{k}[n;l]$ given in Eq. (6), we can show

$$\mathbf{k}[n;l] = (1-\alpha)\mathbf{R}_{\tilde{d}}^{-1}[n;l]\tilde{\mathbf{d}}[n;l].$$

The above equation together with Eq. (8) results in

$$\mathbf{R}_{\tilde{d}}^{H/2}[n;l]\mathbf{k}[n;l] = (1-\alpha)^{1/2}\mathbf{b}_{21}[n;l].$$

Initialization:	
$\mathbf{P}[0;l] = \mathbf{I};$	$0 < \alpha < 1;$
For each n:	
$\pi[n;l] = \mathbf{P}[n-1;l]\tilde{\mathbf{d}}[n;l]$	
$\mathbf{k}[n;l] = \dfrac{\pi[n;l]}{\alpha\cdot(1-\alpha)^{-1} + \tilde{\mathbf{d}}^H[n;l]\pi[n;l]}$	
$\mathbf{P}[n;l] = \alpha^{-1}\mathbf{P}[n-1;l] - \alpha^{-1}\mathbf{k}[n;l]\pi^H[n;l]$	
$(\mathbf{p}[n;l] \mid \star) = \mathbf{P}[n;l]$	
$\hat{d}[n,l] = \tilde{d}[n,l] - \hat{\rho}[n](\mathbf{p}^H[n;l]\tilde{\mathbf{d}}[n;l])$	

Table 3. Algorithm based on the Woodbury identity.

Initialization:		
$\mathbf{P}[0;l] = \mathbf{I};$	$\mathbf{p}[0;l] = \mathbf{e}_1;$	$0 < \alpha < 1;$
For each n:		
$\mathbf{A}[n;l] = \begin{pmatrix} \alpha^{1/2}\mathbf{R}_{\tilde{d}}^{1/2}[n-1;l] & (1-\alpha)^{1/2}\tilde{\mathbf{d}}[n;l] \\ \mathbf{0}^H & 1 \end{pmatrix}$		
$\mathbf{A}[n;l]\Theta[n;l] = \begin{pmatrix} \mathbf{B}_{11}[n;l] & \mathbf{0} \\ \mathbf{b}_{21}^H[n;l] & b_{22}[n;l] \end{pmatrix}$		
$\mathbf{R}_{\tilde{d}}^{1/2}[n;l] = \mathbf{B}_{11}[n;l]$		
$\mathbf{k}[n;l] = (1-\alpha)^{1/2}\{\mathbf{R}_{\tilde{d}}^{-H/2}[n;l]\mathbf{b}_{21}[n;l]\}$		
$\mathbf{p}[n;l] = \alpha^{-1}\mathbf{p}[n-1;l] - \alpha^{-1}\mathbf{k}[n;l](\tilde{\mathbf{d}}^H[n;l]\mathbf{p}[n-1;l])$		
$\hat{d}[n,l] = \tilde{d}[n,l] - \hat{\rho}[n](\mathbf{p}^H[n;l]\tilde{\mathbf{d}}[n;l])$		

Table 4. Algorithm based on QR factorization.

Since $\mathbf{R}_{\tilde{d}}^{H/2}[n;l]$ is upper triangular, $\mathbf{k}[n;l]$ in the above equation can be found via the *back-substitution* algorithm. And finally $\mathbf{p}[n;l]$ in Eq. (7) can be updated.

The algorithm based on QR factorization is summarized in Table 4. This algorithm also supports a fast version of computational complexity of $O(M)$ (see [7]).

4. SIMULATION RESULTS

The OFDM system we simulated in computer employs the parameters given in Table 5. We assumed a number of 4 paths in the *tapped delay line* (TDL) channel model. The paths realizations was selected from a Jakes spectrum [8] with exponential decaying power delay profile. At each simulation run, the path delays were independently and uniformly selected in the interval $[0, T_sN_{\text{cp}} - T_g]$, where is T_s is the sampling period, and the time guard $T_g = 4T_s$ was empirically chosen such that the channel length L satisfies $L \leq N_{\text{cp}} + 1$. For a TDL channel model, Simeone [3] showed that the signal subspace dimension is equal to the number of paths, i.e, $K = 4$. We have not implemented the algorithm for estimating K. The values $K_{\max} = 4$ and $\alpha = 0,99$ were used in the simulation.

The performance of the algorithm based on QR factorization is compared to a low-pass filter of total bandwidth $B = 2f_dT_sN_fM_t$, where f_d is the maximum Doppler frequency and $N_f = N_{\text{cp}} + N_c$. This filter can be performed by $\mathbf{c}[l] = (\mathbf{R} + \delta\mathbf{I})^{-1}\mathbf{r}$, where the ij-th entry of $\mathbf{R}$ is $\text{sinc}(B(i-j))$, $\mathbf{r}$ is the first column of $\mathbf{R}$, and δ is a small constant ($\delta = 10^{-5}$). We selected $M = 41$ for the length of $\mathbf{c}[l]$. As measure of performance we use $\overline{\text{MSE}} = \mathbb{E}\|\mathbf{H}[n] - \hat{\mathbf{H}}[n]\|^2$.

Fig. 2 compares the learning curves of different algorithms for SNR $= 10$ dB, $f_d = 500$ Hz and $M_t = 1$. The $\overline{\text{MSE}}$ was estimated

Sampling rate	800 kHz
Number of subcarriers	$N_c = 128$
Cyclic prefix length	$N_{\mathrm{cp}} = 15$
Number of pilot subcarriers	$N_p = 16$
Pilot symbol constellation type	4-PSK
Data symbol constellation type	16-QAM
Channel power	$\sigma_h^2 = 1$
Symbol power	$\sigma_a^2 = 1$
Number of paths	$K = 4$

Table 5. Simulation parameters.

as the average of 200 realizations. In fact, if compared to a mere subspace projection (legend ST), the filters applied to the subspace components provide more accurate estimates. Although the subspace projections with $\mathbf{U}$ perfectly known (legend U) or estimated by the ST algorithm have similar performance, their versions with filtered subspace components (legends U/Adaptive and ST/Adaptive) attain different $\overline{\mathrm{MSE}}$ values. The observed loss is due to the variability of $\mathbf{U}[n]$. The low-pass filtering in conjunction with the ST algorithm (legend ST/Low-pass) presented the best performance. As drawback, the low-pass approach requires we know the maximum Doppler frequency f_d, whose estimation is not possible in many cases. Now, the proposed adaptive filtering do not demand the estimation of any other parameter.

Fig. 3 show the $\overline{\mathrm{MSE}}$ *versus* SNR curves of different estimators for $f_d = 200\,\mathrm{Hz}$ and $M_t = 3$. For each SNR, the $\overline{\mathrm{MSE}}$ curves are firstly obtained as in Fig 2 for 100 realizations. Subsequently, we found the curves in Fig. 3 as the average of the $\overline{\mathrm{MSE}}$ for iterations from 500 to 1050, where the learning curves achieve the steady state. All curves in Fig. 3 decreases linearly. We got rid of a possible saturation of this curves, differently of which the authors previously found in [5].

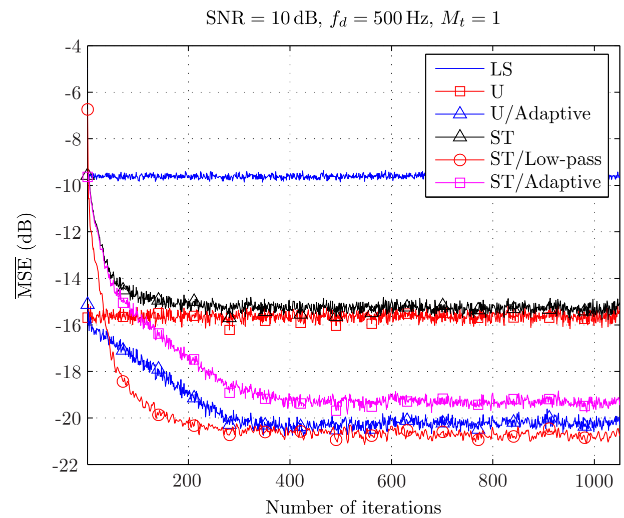

Fig. 2. Learning curves for different algorithms.

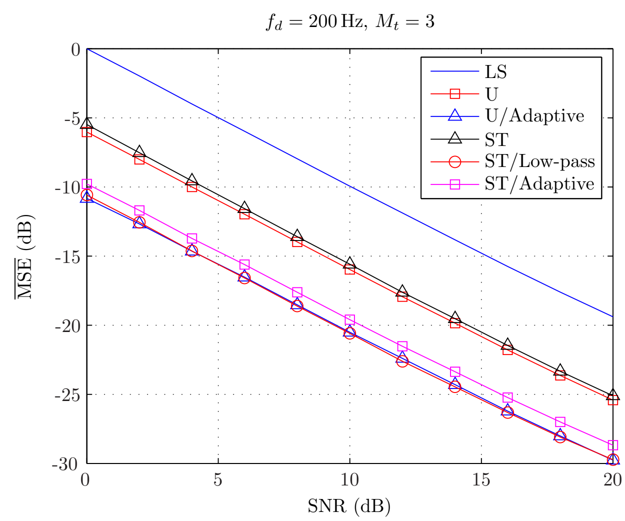

Fig. 3. $\overline{\mathrm{MSE}}$ for different values of SNR.

5. CONCLUSIONS

Based on subspace tracking and QR factorization, we designed an efficient pilot assisted channel estimator for OFDM systems. The QR approach made suitable the simultaneous estimation of $\mathbf{U}$ and $\mathbf{c}[l]$. The adaptive schemes we proposed have good convergence properties and present a linear decaying of the $\overline{\mathrm{MSE}}$ *vs*. SNR curves, as seen in Figs. 2 and 3. The low-pass approach presented best performance. The way we designed the low-pass filter coefficients avoided the saturation found in [5]. Unfortunately, the low-pass approach requires we know the parameter f_d, whose estimation is not possible in many scenarios. On the other hand, the proposed scheme do not require the estimate of any additional parameter. The QR based filtering has the additional advantage of supporting a fast version.

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